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B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2017
FIRST YEAR [BATCH 2016-19]
MATHEMATICS FOR ECONOMICS (General)
Paper : II

Date : 22/05/2017
Time : 11 am – 2 pm
Full Marks : 75

[Use a separate Answer Book for each group]

Group - A

Answer **any seven** questions: [7 X 5]

1. a) State the sequential criterion for existence of limit of a function. 2
   b) Show that if $\underset{x\to a}{Lt}\, f(x)$ exist, then it must be unique. 3

2. a) Give an example of a discontinuous function $f:\mathbb{R}\to\mathbb{R}$, ($\mathbb{R}$ is the set of all real number) which is discontinuous at infinite number of points of $\mathbb{R}$. 2
   b) If $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ are two continuous functions then prove that $f\,g:\mathbb{R}\to\mathbb{R}$ defined by $(f\,g)(x)=f(x)\,g(x)$ is also continuous. 3

3. a) Give an example of a continuous function which is not differentiable. 2
   b) Prove that if $f:\mathbb{R}\to\mathbb{R}$ is differentiable at a point $x = a$, then f must be continuous at a $x=a$. 3

4. a) State Lagrange's mean value theorem. 1
   b) Can we apply L' Hospital's rule to evaluate the following limit? Explain why. 4
   $$\underset{x\to 0}{Lt}\frac{\sin x}{x}$$

5. a) State Rolle's theorem. 3
   b) Can we apply Rolle's theorem on $f(x)=|x|$ in $-1\le x\le 1$. 2

6. a) State the necessary condition for maximum or minimum of a function $y=f(x)$ at any point. 1
   b) Find the maximum and minimum values of $f(x)=1+2\sin x+3\cos^2 x,\ 0\le x\le\frac{\pi}{2}$. 4

7. a) Find $\frac{dy}{dx}$ if $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$. 2
   b) Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t $\tan^{-1}x$. 3

8. a) If $\phi(x)=(x-1)e^x+1$, show that $\phi(x)$ is positive for all positive values of x. 3
   b) If $y=x^{2n}$, where n is a positive integer, show that $\frac{d^n y}{dx^n}=2^n\left\{1\cdot 3\cdot 5\cdots(2n-1)\right\}x^n$. 2

9. a) If $y = \tan^{-1} x$, then prove that

(i) $\left(1+x^2\right) \dfrac{dy}{dx} = 1$ 1

(ii) $\left(1+x^2\right) \dfrac{d^{n+1}y}{dx^{n+1}} + 2nx\dfrac{d^n y}{dx^n} + n(n-1) \dfrac{d^{n-1}y}{dx^{n-1}} = 0$ 4

10. a) Show that the function $f(x) = |x|,\ x \in \mathbb{R}$ has a minimum at $x = 0$. 2

b) Prove that $\underset{x\to\infty}{Lt}\ xe^{-x} = 0$, without using L' Hospital's rule. 3

Group - B

Answer **any four** questions: [4 X 10]

11. a) Define a vector space V over the field F.
Prove that the set of all polynomials is a vector space over the field $\mathbb{R}$ ($\mathbb{R}$ denotes the set of all real numbers). 1+4

b) Find a basis for $\mathbb{R}^4$ that contains the vectors (1, 3, 5, 2) and (3, 2, 4, 6). 5

12. a) Show that $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{R}_{2\times 2} : a+b=0 \right\}$ is a subspace of $\mathbb{R}_{2\times 2}$. Also find a basis for *S*. 5

b) Consider the vector space *V* of all polynomials over the field $\mathbb{R}$. ($\mathbb{R}$ denotes the set of all real numbers).
Prove that the set $\beta = \left\{ x^m : m \text{ is a non-negative integer} \right\}$ is a basis of the vector space *V*. 5

13. For what values of '*a*' the following system of equations is consistent? Also, solve completely in each consistent case. 5+5

$$x + y + z = 1$$
$$2x + 3y - z = a + 1$$
$$2x + y + 5z = a^2 + 1$$

14. a) Consider the system of homogeneous equation *AX* = *O* where *A* is an $n \times n$ matrix and $X = \left[x_1, x_2, \cdots\cdots, x_n\right]^T$. Show that the set of all solutions of this system of equations forms a subspace of $\mathbb{R}^n$. 4

b) Define a linear transformation $T : V \to W$, where *V* and *W* are two vector spaces over the field *F*.
Show that Ker *T* and Im *T* are subspaces of the vector spaces *V* and *W* respectively. 2+4

15. Determine the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ that maps the basis vectors (2, 1, 1), (1, 2, 1), (1, 1, 2) of $\mathbb{R}^3$ to the vectors (1, 1, 1), (1, 1, 1), (1, 1, 1) respectively. Verify that $\dim Ker\, T + \dim \operatorname{Im} T = 3$. 6+4

16. a) Let *V* and *W* be two vector spaces over *F* and $T : V \to W$ be a linear transformation.
If *T* is invertible then show that $T^{-1} : W \to V$ is also a linear transformation. 5

b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(x_1, x_2, x_3) = (x_1 - 3x_2, x_1 + x_2 + 5x_3, x_2 - x_3)$ check whether *T* is invertible and find T^{-1}. (if T^{-1} exists). 5

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